Combining Topological and Geometric Features of Mammograms to Detect Masses

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Abstract. This paper presents a novel method for detecting masses in mammograms. Both topological (salience) and geometric (primarily texture) are used as features to characterise. Experimental results demonstrate that this combination of features is robust both for the segmentation and for the identification of masses.

1 Introduction

Breast cancer is the leading cause of death from cancer among women in many countries. Detecting a breast cancer at the earliest stage possible has the most important impact on prognosis. Mammography is the most cost effective method to detect early signs of breast cancer. However, mammograms are complex, textured images and there is substantial variation across the screening population, leading to difficulties in detection and diagnosis. In addition, the number of mammograms to be analysed in the screening programme is vast (eg 3 million annually in the UK alone). This is the motivation for the development of computer-aided diagnosis (CAD) systems that provide a consistent and reproducible second opinion to a radiologist. Unfortunately, progress toward building CAD systems for mass detection has been considerably slow, not least because of the subtle characteristics of their appearance [1]. While most previous work has been based on pixel-based statistical approaches, we aim to delineate regions corresponding to masses by analysing both the topological and geometrical structure of an image. Based, initially on regions of interest extracted by the algorithm presented in our previous work [2], we develop in this paper a two stage strategy. First, the mammogram is segmented into candidate mass regions based on the mammogram topology. This stage selects all salient regions that include masses along with a number of false regions that are candidates for masses. Then the segmented candidate mass regions are classified as masses or rejected on the basis of a measure of topological saliency combined with geometric texture features.

2 Segmentation

The first stage of our method is to segment the mammogram into salient regions which include all mass regions but also a small number of normal dense bright regions which constitute false positives. A segmentation method using topological properties of image structure has been presented and an efficient implementation is also provided.

2.1 Salient Region

Masses generally appear to be dense bright regions. The algorithm for detecting masses initially delineates the boundary of salient regions characterised by intensity blobs. The proposed segmentation algorithm analyses the topological structure of an image to detect salient regions as candidate masses. In analysing image structure, an image $I(\mathbf{x}), \mathbf{x} \in \Omega \subset \mathbb{R}^2$ is considered as a surface $\phi : \Omega \to \mathbb{R}^+$ where the intensity is regarded as the height of the surface. A region $R(t)$ and a level curve $\Gamma(t)$ at a level t on a surface ϕ is defined by:

$$\begin{aligned} R(t) &= \{\mathbf{x} | \phi(\mathbf{x}) > t\} \\ \Gamma(t) &= \partial R(t) \end{aligned}$$

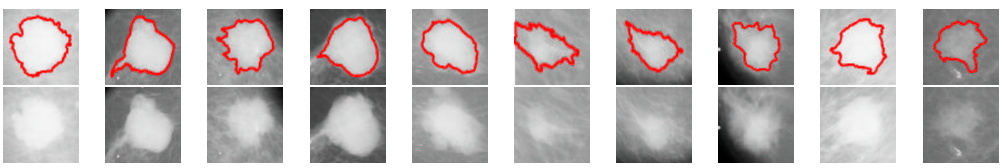
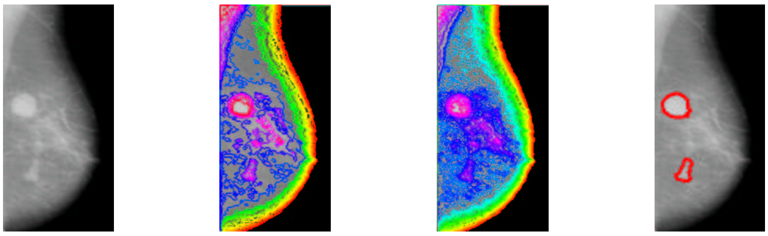


Figure 1. Examples for the segmentation of topologically salient regions. Top: Most salient base contours superimposed on the mammograms. Bottom: Original mammograms.



(a) (b) (c) (d)

Figure 2. (a) A mammogram. (b) Coarse-scale contour map (N=30). (c) Fine-scale contour map (N=100). (d) Detected salient regions after discarding contours for the breast boundary and the pectoral muscle from (c).

A level curve is constrained to be a simple closed curve, thus the complement of a curve Γ consists of exactly two disjoint regions, an interior and an exterior. By definition of the level curve, a surface within an interior region of the level curve always forms a blob region that is higher than its local neighbourhood. In observing a topological change of a surface, saddle points and local maximum points are of interest. Let H(ϕ) be the Hessian of a surface ϕ and det(H(ϕ)) be its determinant. The set of all local maximum points on a surface ϕ within a region R is given by:

$$\mathbf{P}_M(R) = \{\mathbf{x} | \nabla\phi(\mathbf{x}) = 0, \det(\mathrm{H}(\phi)) > 0, \frac{\partial^2\phi}{\partial x^2} < 0, \frac{\partial^2\phi}{\partial y^2} < 0, \forall \mathbf{x} \in R\}$$

Similarly, the set of all saddle points on a surface ϕ within a region R is given by:

$$\mathbf{P}_S(R) = \{\mathbf{x} | \nabla\phi(\mathbf{x}) = 0, \det(\mathrm{H}(\phi)) < 0, \forall \mathbf{x} \in R\}$$

A saddle point implies a topological change of a surface, since the split of an object occurs at a saddle point as the level is elevated. For a given image $I(\mathbf{x})$, all saddle points $\mathbf{P}_S(\Omega)$ on a surface ϕ are initially determined for segmenting salient regions. Once the saddle points are known, all level curves at saddle points, called the *base level curves*, are given as:

$$\Gamma^* = \{\Gamma_i | \Gamma_i = \partial R_i(t), t = I(p), \forall p \in \mathbf{P}_S\}$$

Then, each base level curve $\Gamma_i \in \Gamma^*$ provides segmentation for the boundary of a candidate salient region R_i. The details of the segmentation algorithm are described in [2].

3 Region Classification

The list of mass candidate regions defined by the base level curves includes all real masses together with a number of false positives. In order to reduce the false positive rate of the system, features that are important for identifying masses are developed based on topological saliency measures and geometrical texture features. These features are used for classifying salient regions obtained by the segmentation into masses or normal dense tissues.

3.1 Topological Saliency Measures

Structural Contrast : A structure-based contrast measure is proposed for measuring the saliency of a region. The structural contrast of a region is defined as the difference between the intensity of a local maximum point within the region and the intensity of the base level curve that delineates that region. The minimum structural contrast $K(\Gamma(t))$ for a base level curve $\Gamma(t)$ on an image surface $\phi(\mathbf{x}, I(\mathbf{x}))$ is defined as:

$$K(\Gamma(t)) = \min_{\mathbf{x}_i \in \mathbf{P}_M(\Gamma(t))} (I(\mathbf{x}_i) - t)$$

If there is more than one local maximum within the region, the minimum contrast is defined so that it also takes into account the variance of the image structure.

Structural Variety : The structural variety implies a topological saliency in that a salient region is likely to be formed by one object that leads the region to have one local maximum. The structural variety $V(\Gamma(t))$ for a base level curve $\Gamma(t)$ is given by the number of local maximum points within $Int(\Gamma(t))$ as follows:

$$V(\Gamma(t)) = \frac{|\mathbf{P}_M(\Gamma(t))|}{A(\Gamma(t))}$$

where $A(\Gamma(t))$ is the area of the region within $\Gamma(t)$. The larger regions will naturally contain more local maximum points, so the measure for the structural variety is normalised by the area.

3.2 Geometric Texture Features

The texture features used are amplitude, contrast and orientation invariant local geometrical descriptions of the intensity surface. These provide a rich description of the image surface that complements the topological features. The features are derived from dimensionless combinations of linear filter responses. The basis for these filters is a rotationally symmetric filter $f(r)$. From this filter, further filters are generated by multiplying $f(r)$ by angular trigonometric functions:

$$\begin{aligned} h_n(r,\theta) &= f(r) * \cos(n\theta) \\ l_n(r,\theta) &= f(r) * \sin(n\theta) \end{aligned}$$

where $r = \|\mathbf{x}\|_2$ and $\theta = \tan^{-1}(x_1/x_2)$. These filters convolved individually with the image regions. A family of contrast invariant local symmetry descriptors can be generated from the responses to these filters as follows:

$$\Phi_n(\mathbf{x}) = \tan^{-1} \frac{(f \otimes I(\mathbf{x}))}{\sqrt{(h_n \otimes f \otimes I(\mathbf{x}))^2 + (l_n \otimes f \otimes I(\mathbf{x}))^2}}$$

These descriptors have values in the range $[0, \pi]$. Φ_1 is equivalent to the phase of the monogenic signal [3], and describes odd-even local symmetry. Φ_2 is related to the shape index [4] (if f_r is a Laplacian of a Gaussian, they are equivalent) and describes local even-even symmetry (i.e. ridges, troughs, saddle points and local maxima and minima). The local symmetry descriptors are each accompanied by an orientation. The offset between the orientation of each symmetry and the local principal orientation provides an additional local descriptor. The local orientations are encoded by phasors:

$$O_n(\mathbf{x}) = h_n(\mathbf{x}) + i \cdot l_n(\mathbf{x})$$

The angle of each phasor is a measure of the local orientation of each symmetry descriptor. ($\theta_n = \tan^{-1}(h_n/l_n)/n$). For the results presented in this paper, only the first three descriptors were used. The local orientations of each descriptor are normalised by dividing by a local orientation phasor:

$$\begin{aligned} O_m(\mathbf{x}) &= \frac{O_1(\mathbf{x})^3}{\|O_1\|^2} + \frac{O_2(\mathbf{x})^2}{\|O_2(\mathbf{x})\|} + O_3(\mathbf{x}) \\ O_n'(\mathbf{x}) &= \frac{O_n(\mathbf{x})\|O_m(\mathbf{x})\|}{O_m(\mathbf{x})\|O_n(\mathbf{x})\|} \end{aligned}$$

This gives six invariant descriptors, Φ_{1-3} and O'_{1-3}. The filters used here are scale and affine robust:

$$f(r) = \frac{A}{r^{\alpha+\beta}} - \frac{B}{r^{\alpha-\beta}}$$

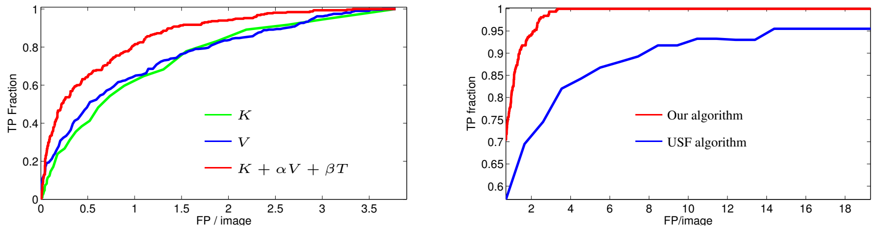
Two members of this family were used, giving twelve descriptors in all. The two filters are given by $\alpha = [2.75, 3.75]$, $\beta = 0.25$, with A and B chosen to give a zero DC component (that is, provide a measure that is invariant to brightness). Textures are characterised by the distribution of energy over these descriptors, and in particular the co-occurrence of energy in the different descriptors. It is impractical to learn the co-occurrence of all descriptors because of the high dimensionality. Instead, the joint energy distributions of each possible pair were estimated, giving a total of 66 two-dimensional distributions. The energy distributions are calculated as follows. In order to learn characteristic features for masses, the training set consists of a list of mass candidates, each manually labelled as belonging to one of two classes, true positive or false positive. For all mass candidates in a class, the six descriptors described above are calculated, along with the local energy (the square root of the sum of squares of all thirteen filter responses at each point). For each of the 66 possible descriptor pairings, a local energy weighted histogram is computed, the set of which acts as a region texture descriptor. The variance of each histogram location is also estimated. To classify, a variance weighted energy distribution difference is calculated:

$$E_{\text{diff}} = \frac{E_{FP} - E_{TP}}{\sqrt{V_1 + V_2 + \frac{1}{M}}}$$

where M is the number of histogram bins, here 50x50 histograms were used (N=2500). The $1/M$ term is added to reduce the influence of parts of the histogram which have had very few votes. For each unclassified mass candidate the energy distribution, $E(\Gamma(t))$ is computed as above. The region is then given a score, $T(\Gamma(t))$, by taking the inner product of its energy distribution with E_{diff}:

$$T(\Gamma(t)) = \sum E_n * E_{\text{diff}}$$

$T(\Gamma(t))$ may be treated as a "texture saliency" measure, to complement the topological saliency measures.



(a) (b)

Figure 3. (a) ROC operated by different saliency measures. (b) ROC comparison with the USF algorithm.

4 Experiments

We present experimental results based on a set of 400 mammograms with masses varying in size and subtlety. Mammograms were selected from various pathological categories in the USF database [5]. The resolution of the data set was 200μm per pixel. The detection of masses was determined by the ratio of the intersection between a segmented region and the ground truth over the union of them. In the segmentation stage, a coarse-scale contour map ($N = 30$) was initially used to localise topologically salient regions characterised by base contours. Among the salient regions, the breast boundary and the pectoral muscle were excluded by using their strong anatomical constraints in terms of size and location. Then, the salient contours that correspond to the ones in the coarse-scale contour map were identified in a fine-scale contour map ($N = 100$) by traversing inclusion tree. The topological saliencies were measured based on the fine-scale contour map and the geometric texture descriptor was measured based on the interior regions of salient contours selected from the fine-scale contour map. More accurate both segmentation and saliency measurement were obtained from the fine-scale contour map. An example for contour maps with different scales and extracted base contours are shown in Figure 2. The base contours with high minimum structural contrast and low structural variety define the boundaries of salient regions for mass candidates. The algorithm achieved 100% detection rate with 3.3 false positives per image and 90% with 1.4 respectively. The performance of the algorithm was evaluated by FROC operated by the saliency measure, K, V, and $K + \alpha V + \beta T$ with the optimal choice of weights as presented in Figure 3(a). We have used no priori information about mass at measuring K and V, but we have trained textures of mass using 100 samples. An additional texture information to topological saliency improved the results as shown in Figure 3(a). The comparison of our results with the USF algorithm [5] is presented in Figure 3(b). In addition to the improved performance, our algorithm provides exact boundary of masses as opposed to just finding a point within the mass region.

5 Conclusion

We have developed a blend of topological and geometrical descriptors that together effectively characterise mass features. A global structural approach based on a topographic representation is shown to be a useful counterpart to a local statistical approach for detecting masses in mammograms. Structural saliency measures are shown to be useful mammographic features to delineate regions of interest and the texture information using phase information appears to be an informative characteristic feature of mass. Experimental results indicate that this method can be used as the basis for an effective prompting tool to assist radiologist in the diagnosis of breast cancer.

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