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Region matching with missing parts☆

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Abstract

We present a variational approach to the problem of registering planar shapes despite missing parts. Registration is achieved through the evolution of a partial differential equation that simultaneously estimates the shape of the missing region, the underlying 'complete shape' and the collection of group elements (Euclidean, affine) corresponding to the registration. Our technique can be used both for shapes, for instance represented as characteristic functions (binary images) and for grayscale images where it can be interpreted as region 'inpainting.' The novelty of the approach lies on the fact that, rather than estimating the missing region in each image independently, we pose the problem as a joint registration with respect to an underlying 'complete shape' from which the complete version of the original data is obtained via a group action. We simultaneously estimate the complete shape and the group action in an alternating minimization scheme.

Keywords: Shape; Variational; Registration; Missing Part; Inpainting

1. Introduction

Consider different images of the same scene, taken for instance from a moving camera, where one or more of the images have been corrupted, so that an entire part is missing. This problem arises, for instance, in image registration with missing data, in the presence of occlusions, in shape recognition when one or more parts of an object may be absent in each view, and in 'movie inpainting' where one or more frames are damaged and one wants to 'transfer' adjacent frames to fill in the damaged part.

We consider a simplified version of the problem, where we have a compact region in each image i, bounded by a closed planar contour, γ_i, and a region of the image, with support described by a characteristic function χ_i, is damaged. We do not know a-priori what the region χ_i is, and we do not know the transformation mapping one image onto the other. However, we make the assumption that such a transformation can be well approximated by a finite-dimensional group g_i, for instance the affine or the projective group. In addition, we do not know the value of the image in the missing region. Therefore, given a sequence of images, one has to simultaneously infer the missing regions χ_i as well as the transformations g_i and the occluded portions of each contour γ_i.

We propose an algorithm that stems from a simple generative model, where an unknown contour μ_0, the 'complete shape,' is first transformed by a group action g_i, and then occluded by a region χ_i. Therefore, one only estimates the complete shape and the group actions, thereby rendering the problem well-posed

$$\hat{g}_1,\ldots,\hat{g}_k,\hat{\mu}_0,\hat{\chi}_1,\ldots,\hat{\chi}_k = \arg\min_{g_i,\mu_0} \sum_{i=1}^{k} \phi(\chi_i(\gamma_i), \chi_i \circ g_i(\mu_0)) \tag{1}$$

for a given discrepancy measure ϕ. A simpler case is when the occlusion occurs at the same location in all shapes; in this case, there is only one indicator function χ_0 that acts on the complete shape μ_0.

2. Background and prior work

The analysis of 'Shape Spaces' was pioneered in Statistics by Kendall, Mardia and Carne among others [15, 22,9,25]. Shapes are defined as the equivalence classes of points modulo the similarity group, $\mathbb{R}^{MN}/\mathrm{SE}(M)\mathbb{R}$. The equivalence class induces a fiber bundle structure where

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motion along the fibers corresponds to the alignment of the collection of points, while motion across fibers corresponds to deformations. A metric and the associated probability densities can be defined on the base of the fiber bundle, which allows comparing so-defined shapes and characterize the uncertainty around a shape in a way that is invariant with respect to the similarity group. Invariance to the group is achieved by projection onto the base of the fiber bundle, which can be done in closed-form for the case of the similarity group. These tools have proven useful in contexts where N distinct 'landmarks' are available, for instance in comparing biological shapes with N distinct 'parts.' Various extensions to missing points have been proposed, mostly using expectation-maximization (EM) and alternating between computing the sufficient statistics of the missing data and performing shape analysis in the standard framework of shape spaces. However, since this framework is essentially tied to representing shapes as collections of points, they do not extend to the case of continuous curves and surfaces in a straightforward way, and we will, therefore, not pursue them further here.

In computational vision, a wide literature exists for the problem of 'matching' or 'aligning' discrete representations of collections of points, for instance organized in graphs or trees [21,11]. A survey of shape matching algorithms is presented in [36]. Just to highlight some representative algorithms, Belongie et al. [5] propose comparing planar contours based on the 'shape context' of each point along the contour. Although this is still a point-based approach, points are not bound to represent particular 'landmarks' but are just a discrete representation of the contour. Their matching is, by construction, invariant with respect to the affine group, and is somewhat robust to missing parts since the shape context changes moderately even in the presents of large changes in the underlying images. This work is, therefore, positioned somewhere in between landmark or feature-based approaches and image-based ones, similarly to [10]. Koenderink [20] is credited with providing some of the key ideas involved in formalizing a notion of shape that matches the intuitive notion. Mumford has critiqued current theories of shape on the grounds that they do not capture the essential features of perception [27].

'Deformable Templates', pioneered by Grenander [12], do not rely on a point-wise representation; rather, images are deformed under the action of a deformation group (possibly infinite-dimensional) and compared for the best match in an image-based approach [39,6]. Grenander's work sparked a current that has been particularly successful in the analysis of medical images, for instance [13]. In this work we would like to retain some of the features of deformable templates, but extend them to modeling missing parts.

A somewhat different line of work is based on variational methods and the solution of partial differential equations (PDEs) to deform planar contours and quantify their 'distance.' Not only can the notion of alignment or distance be made precise [4,38,26,18,32], but quite sophisticated theories of shape, that encompass perceptually relevant aspects, can be formalized in terms of the properties of the evolution of PDEs (e.g. [19,16]). The variational framework has also been proven very effective in the analysis of medical images [24,35,23]. Zhu et al. [40] have also extended some of these ideas to a probabilistic context.

A common approach to matching planar contours within the context of scale-space is to not match the contours directly, but to first represent them in a common scale-space and then matching a given scale, or even all scales. The rationale being that, even if the original contours are not well matched by a group action, their scale-space representation at some level may. Scale-space is a very active research area, and some of the key contributions as they relate to the material of the present paper can be found in [14,34,17,2,3,1] and references therein (Fig. 1).

Other techniques rely on matching different representations, for instance skeletons [16], that are somewhat robust to missing parts. In [33] a similar approach is derived using a generic representation of 2-D shape in the form of structural descriptions from the shocks of a curve evolution process, acting on bounding contours. The shocks are organized into a graph, and complexity is managed by attending to the most significant shape components first. The problem of shape matching is faced after a reduction of a shock graph to a unique rooted shock tree by the shock graph grammar rules.

In region-based registration, one assumes identical objects (i.e. objects equivalent under the Euclidean group), and finds the motion mapping one onto the other by matching image statistics.

The possibility of making multiple registration by finding a mean shape and a rigid transformation was studied by Pennec [30] in the case of 3D landmarks. In this paper, applications to synthetic and real experiments in molecular biology and medical imaging are presented.

Leung, Burl and Perona [8] described an algorithm for locating quasi-frontal views of human faces in cluttered scenes that can handle partial occlusions. The algorithm is based on coupling a set of local feature detectors with a statistical model of the mutual distances between facial features.

In this work, we intend to extend these techniques to situations where part of the image cannot be used for matching (see [37]) and at the same time landmark approaches fail.

In the paper of Berger and Gerig [7], a deformable area-based template matching is applied to low contrast medical images. In particular they use a least squares template matching (LSM) with an automatic quality control of the resulting match.

Nastar, Moghaddam and Pentland [28] proposed to use a statistical learning method for image matching and interpolation of missing data. Their approach is based on the idea of modeling the image like a deformable intensity surface represented by a 3D mesh and use principal

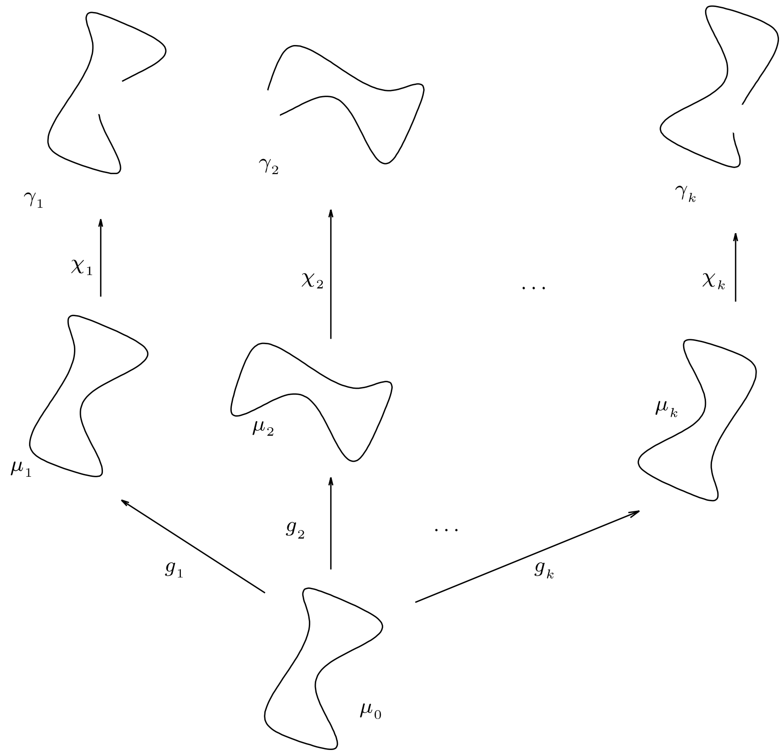

Fig. 1. A contour undergoes a global motion and local occlusions.

component analysis to provide a priori knowledge about object-specific deformations.

Rangarajan, Chui and Mjolsness [31], defined a novel distance measure for non-rigid image matching where probabilistic generative models are constructed for the nonrigid matching of point-sets via an explicit Platonist formulation.

2.1. Contributions of this paper

This work presents a framework and an algorithm to match regions based on image statistics despite missing parts. To the best of our knowledge, work in this area, using region-based variational methods, is novel. Our framework relies on the notion of 'complete shape' which is inferred simultaneously with the group actions that map the missing shape onto the incomplete ones. The complete shape and the registration parameters are defined as the ones that minimize a variational cost functional, and are computed using an alternating minimization approach where a partial differential equation is integrated using level set methods.

3. Matching with missing parts

The formulation of the problem and the derivation of the evolution equations are introduced in this section for the case of a planar shape under isometric transformation (rigid motions). In this case, the dimension of the space is 2 and the determinant of the Jacobian of the group is $J(g)=1$ for all the elements g of the group G. The main assumption about the shape is that it must be a *regular domain*, that is an open and bounded subset of $\mathbb{R}^2$ with a finite number of connected components and a piecewise C^∞ boundary. This regularity is required to avoid singular pathologies and to make all the computation possible. The main notation that will be used in this section is listed below.

Notation

$\bar{\gamma}_i$, $\bar{\mu}$, regular domains in $\mathbb{R}^2$
γ_i, μ, the boundaries of $\bar{\gamma}_i$, $\bar{\mu}$
$\chi(\gamma)$, the characteristic function of the set $\bar{\gamma}$
$A(\gamma)$, the area (volume) of the region $\bar{\gamma}$
$\langle\cdot,\cdot\rangle$, the usual inner product

3.1. Formulation of the problem

Let $\bar{\gamma}_1, \ldots, \bar{\gamma}_k$ be regular domains of $\mathbb{R}^2$, all obtained from the same regular domain $\bar{\mu} \subset \mathbb{R}^2$ by multiplication by characteristic functions

$$\chi_1, \ldots, \chi_k : \mathbb{R}^2 \to \mathbb{R}$$

and actions of Lie Group elements $g_1,\ldots,g_k \in G$. We want to find the best solution in the sense expressed by the functional

$$\phi = \sum_{i=1}^{k} A(\bar{\gamma}_i \backslash g_i(\bar{\mu})) + \alpha A(\bar{\mu}) \quad (2)$$

where A denotes the area, α is a constant, $\bar{\mu}$, χ_i and g_i are the unknowns and the sets $\bar{\gamma}_i$ and the structure of G are given.

3.2. Derivative with respect to the shape

The functional ϕ can be written in integral form

$$\phi = \sum_{i=1}^{k} \int_{\bar{\gamma}_i} (1 - g_i\bar{\mu})\mathrm{d}x + \alpha \int_{\bar{\mu}} \mathrm{d}x \tag{3}$$

and using the characteristic function notation

$$\phi = \sum_{i=1}^{k} \int \chi(\gamma_i)(1 - \chi(g_i\mu))\mathrm{d}x + \alpha \int \chi(\mu)\mathrm{d}\mathbf{x} \tag{4}$$

$$\phi = \sum_{i=1}^{k} \int \chi(\gamma_i)\mathrm{d}x - \sum_{i=1}^{k} \int \chi(\gamma_i)\chi(g_i\mu)\mathrm{d}x + \alpha \int \chi(\mu)\mathrm{d}x \tag{5}$$

Since the first term of ϕ is independent of μ, g and remembering that g_i are isometries, the problem of minimizing ϕ is equivalent to that of finding the minimum of the energy

$$E(g_i, \mu) = \int_{\bar{\mu}} \left(\alpha - \sum_{i=1}^{k} \chi(g_i^{-1}\gamma_i) \right) \mathrm{d}\mathbf{x}. \tag{6}$$

One can show that the first variation of this integral along the normal direction of the contour is simply its integrand, by using the divergence theorem, and therefore conclude that a gradient flow that minimizes the energy E with respect to the shape of the contour μ is given by

$$\frac{\partial \mu}{\partial t} = \left(\alpha - \sum_{i=1}^{k} \chi(g_i^{-1}\gamma_i) \right) N \tag{7}$$

where N is the normal vector field of the contour μ.

3.3. Derivative with respect to the group actions

In order to compute the variation of the functional ϕ with respect to the group actions g_i, we first notice that there is only one term that depends on g_i in Eq. (6). Therefore, we are left with having to compute the variation of

$$-\int_{\bar{\mu}} \chi(g_i^{-1}\gamma_i)\mathrm{d}\mathbf{x}. \tag{8}$$

In order to simplify the notation, we note that the term above is of the generic form

$$W(g) \doteq \int_{\bar{\mu}} f(g(\mathbf{x}))\mathrm{d}\mathbf{x} \tag{9}$$

with $f = \chi(\gamma_i)$. Therefore, we consider the variation of W with respect to the components of the exponential coordinates[1] ξ_i of the group $g_i = e^{\hat{\xi}_i}$

$$\frac{\partial W}{\partial \xi_i} = \frac{\partial}{\partial \xi_i} \int_{\bar{\mu}} f(g(\mathbf{x}))\mathrm{d}\mathbf{x} \tag{10}$$

$$\frac{\partial W}{\partial \xi_i} = \int_{\bar{\mu}} \nabla f(g(\mathbf{x})) \frac{\partial}{\partial \xi_i} g(\mathbf{x})\mathrm{d}\mathbf{x}. \tag{11}$$

Using Green's theorem it is possible to write the variation as an integral along the contour of μ

$$\frac{\partial W}{\partial \xi_i}(g) = \int_{\mu} f(g(\mathbf{x})) \left\langle \frac{\partial \hat{\xi}}{\partial \xi_i} g, N \right\rangle \mathrm{d}s. \tag{12}$$

Therefore, the derivative with respect of the group action is

$$\frac{\partial \phi}{\partial \xi_i} = -\int_{\mu} \chi(g_i^{-1}\bar{\gamma}_i) \left\langle \frac{\partial \hat{\xi}_i}{\partial \xi_i} g_i, N \right\rangle \mathrm{d}s. \tag{13}$$

3.4. Evolution equations

Within the level set framework a function ψ is evolved instead of the contour μ. The function ψ is negative inside μ, positive outside and zero on the contour. The evolution of ψ depends on the velocity of μ via the Hamilton–Jacobi equation

$$\begin{cases} u_t + \left(\alpha - \sum_{i=1}^{k} \chi(g_i^{-1}\gamma_i) \right) |\nabla u| = 0, \\ u(0, x) = \psi^{(t)}(x). \end{cases} \tag{14}$$

The evolution equations follow

$$\psi^{(t+1)}(x) = u(1, x|\psi^{(t)}) \tag{15}$$

$$\bar{\mu}^{(t)} = \{x : \psi(x) < 0\} \tag{16}$$

$$\xi_i^{(t+1)} = \xi_i^{(t)} + \beta_\xi \int_{\mu} \chi(g_i^{-1}\bar{\gamma}_i) \left\langle \frac{\partial \hat{\xi}_i}{\partial \xi_i} g_i, N \right\rangle \mathrm{d}s \tag{17}$$

where β_ξ is a step parameter and $u(\cdot, \cdot|\psi^{(t)})$ is the solution of problem (14) with initial condition $u(0, x) = \psi^{(t)}(x)$.

3.5. Generalization to graylevel images

There are many possible generalizations of the above formulas. Here, we present a brief generalization to the case of gray level images. The case of color images is very

[1] Every finite-dimensional Lie group admits exponential coordinates. For the simple case of the isometries of the plane, the exponential coordinates can be computed in closed-form.

similar so the equations will not be stated. The main idea is to work with the level set of each level between the minimum and the maximum of the images and write the functional to match all the level sets. Let k images I_i of the same object be given and

$$\bar{\gamma}_i^j = \{x : I_i < j\delta\} \tag{18}$$

be the $j\delta$ underlevel of I_i, where δ is a small discretizing term and $j=1,\ldots,n$ is the number of levels. This time let

$$\mu : \mathbb{R}^2 \to \mathbb{R} \tag{19}$$

be a function that represents the complete image and

$$\bar{\mu}^j = \{x : \mu < j\delta\} \qquad \mu^j = \partial\bar{\mu}^j \tag{20}$$

respectively, its underlevel and the boundary of the underlevel. Using the same consideration developed for the case of a contour, we write the functional ϕ as

$$\phi = \sum_{i=1}^{k}\sum_{j=1}^{n} A(\bar{\gamma}_i^j \backslash g_i(\bar{\mu}^j)) + \alpha \int_{j=1}^{n} A(\mu^j) \tag{21}$$

$$\phi = \sum_{i=1}^{k}\sum_{j=1}^{n} \int \chi(\gamma_i^j)(1-\chi(g_i\mu^j))\mathrm{d}x + \alpha \sum_{j=1}^{n} \chi(\mu^j)\mathrm{d}x \tag{22}$$

and in the same way the derivatives of the functional can be obtained as

$$\frac{\partial \mu^j}{\partial t} = \left(\alpha - \int_{i=1}^{k} \chi(g_i^{-1}\gamma_i^j)\right) N^j, \tag{23}$$

$$\frac{\partial \phi}{\partial \xi_i} = -\int_{j=1}^{n} \int_{\mu^j} \chi(g_i^{-1}\gamma_i^j) \left\langle \frac{\partial g_i}{\partial \xi_i}, N^j \right\rangle \mathrm{d}s \tag{24}$$

Eq. (23) gives the Hamilton–Jacobi equation for the function μ

$$\mu_t(t,x) + \left(\sum_{i=1}^{k} H(I_i(g_i(x)) - \mu(t,x)) - \alpha\right) |\nabla\mu(t,x)| = 0 \tag{25}$$

where

$$H(s) = \begin{cases} 1 & \text{if } s > 0, \\ 0 & \text{if } s \leq 0. \end{cases} \tag{26}$$

Therefore, the evolution equation for both the function μ and the parameter of g_i are stated as

$$\mu^{(t+1)}(x) = \mu^{(t)}(x) - \beta_\mu \left(\int_{i=1}^{k} H(I_i(g_i^{(t)}(x)) - \mu^{(t)}(x)) - \alpha \right) |\nabla\mu^{(t)}(x)| \tag{27}$$

$$\xi_i^{(t+1)} = \xi_i^{(t)} + \beta_\xi \int H(I_i(g_i^{(t)}) - \mu^{(t)}) \left\langle \frac{\partial g_i^{(t)}}{\partial \xi_i^{(t)}}, \nabla\mu^{(t)} \right\rangle \mathrm{d}x, \tag{28}$$

where β_μ and β_ξ are a step parameters.

4. Experiments

A numerical implementation of the evolution Eq. (15–17) has been written within the level set framework proposed by Osher and Sethian [29] using the Ultra Narrow Band algorithm and an alternating minimization scheme. A set of common shapes (hands, leaves, mice, letters) has been chosen and converted into binary images of 256×256 pixels. For each image of this set a group of binary images with missing parts and with a different pose on the plane has been generated (Figs. 2 and 3 curves γ_1, γ_2).

The following level set evolution equation has been used

$$\mu_t + \left(\alpha - \int_{i=1}^{k} \chi\left(g_i^{-1}\gamma_i\right)\right) |\nabla\mu| = 0 \tag{29}$$

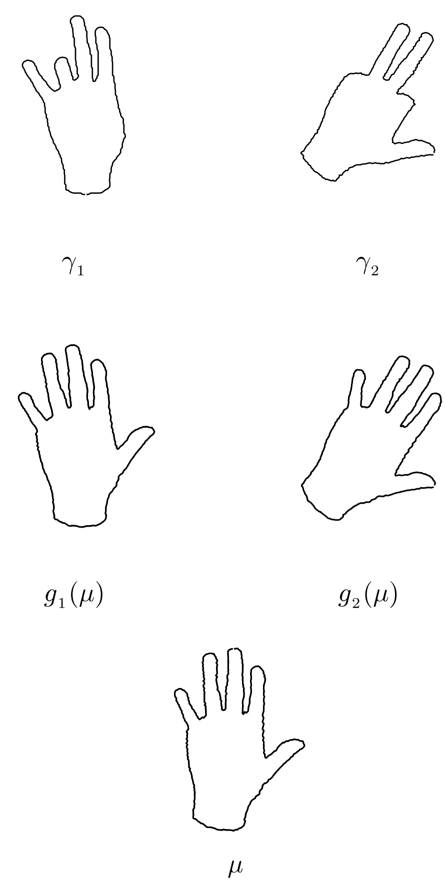

Fig. 2. Hands (top) a collection of images of the same hand in different poses with different missing parts. The support of the missing part is unknown, (middle) similarity group, visualized as a ‘registered’ image, (bottom) estimated template corresponding to the similarity group (‘complete shape’).

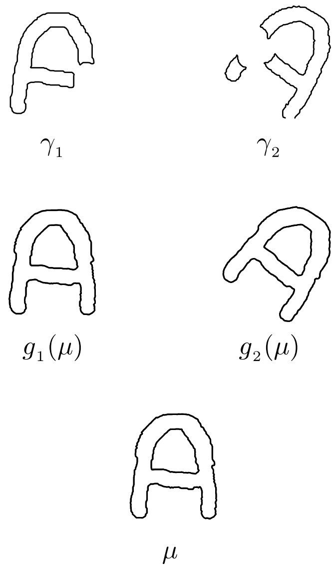

Fig. 3. Letter ‘A’ (top) a collection of images of the letter A in different poses with different missing parts. The support of the missing part is unknown (middle) similarity group (‘registration’), (bottom) estimated template corresponding to the similarity group (‘complete shape’).

with a first-order central scheme approximation. The evolution of the pose parameters has been carried out using the integral (17) with the following approximation of the arclength ds

$$ds \approx |\nabla(\mu)| d\mathbf{x}. \tag{30}$$

The evolution has been initialized with the following settings

$$\mu_{t=0} = \gamma_1 \qquad T_i = B_{\gamma_i} - B_{\gamma_1}$$
$$R_i = \begin{pmatrix} \cos\theta_i & -\sin\theta_i \\ \sin\theta_i & \sin\theta_i \end{pmatrix}, \quad \text{with } \theta_i = \widehat{E_{\gamma_i} O E_{\gamma_1}} \tag{31}$$

where β_{γ_i} is the baricenter of γ_i and E_{γ_i} is the principal axis of inertia of the region $\bar{\gamma}_i$. The value of α has been set between 0 and 1.

In Figs. 2 and 3 some results have been illustrated. γ_1 and γ_2 are the starting curves and μ is the complete shape in an absolute system after the computation. The figures show the computed $g_1(\mu)$ and $g_2(\mu)$ the estimated rigid motions.

Fig. 4 shows results of the matching of a collection of images of the corpus Callosum of different patients. It

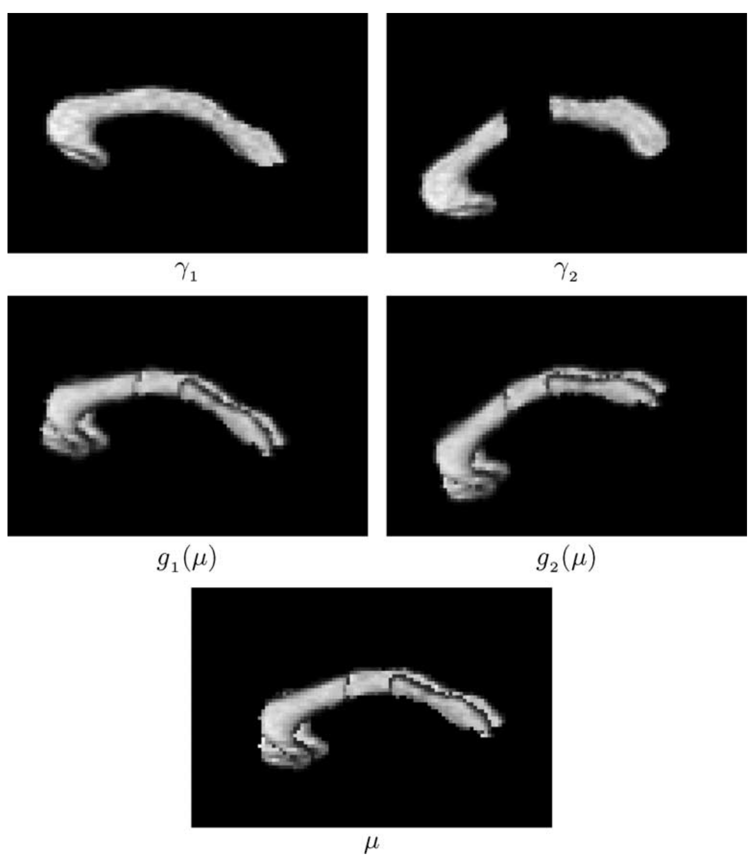

Fig. 4. Corpus Callosum (top) a collection of images of the Corpus Callosum in different poses with different missing parts. The support of the missing part is unknown (middle) registration, (bottom) estimated template corresponding to the similarity group (‘complete shape’). Note that the original shapes were not equivalent, in the sense that they belong to two different subjects and therefore there is no rigid motion that brings one exactly onto the other. Nevertheless, the registration is accurate despite the missing part. One way to improve is to consider richer groups than simple rigid motions. The derivation and the analytical techniques used are essentially the same.

should be noticed that this case does not fall strictly within the framework discussed in this paper. In fact, the two shapes are not equivalent, in the sense that there is no group action G that maps one into the other, due to the deformation. Nevertheless, the alignment is quite compelling despite the missing part. One way to further improve this technique is to use richer finite-dimensional groups G that can account for more than simple rotations and translations. Simple examples include the affine and projective groups, for which the derivation is essentially the same, except for a few changes of measure due to the fact that they are not isometric.

The experiments show that this method works very well even when the missing part in each image is pretty significant, up to about 20% of the area. Generalization to k shapes is straightforward.

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