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A Fast RANSAC–Based Registration Algorithm for Accurate Localization in Unknown Environments using LIDAR Measurements

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Abstract— **The problem of accurate localization using only measurements from a LIDAR sensor is analyzed in this paper. The sensor is rigidly fixed on a generic moving platform, which moves on a plane. Practical on–line applications of localization algorithms impose constraints on the execution time, problem that is addressed in this paper and compared with other existing solutions.**

Due to the nature of the sensor adopted, the localization algorithm is based on a fast and accurate *registration* algorithm, which is able to deal with noisy measurements, outliers and dynamic environments. The proposed solution relies on the RANSAC algorithm in combination with a Huber kernel in order to cope with typical nuisances in LIDAR measurements. The robust registration is successively used in combination with an Extended Kalman Filter to track the trajectory of the LIDAR over time, hence to solve the localization problem.

Simulations and experimental results are reported to show the feasibility of the proposed approach.

I. INTRODUCTION

One of the main concerns for autonomous mobile robots is their ability to *localize* w.r.t. a fixed reference frame as they travel in an environment. In literature, the *localization* problem is related to the robot's position estimation in a mapped environment, problem also referred in literature as *kidnapped robot problem* ([1]), and to the maintenance of positions information over time. Since the localization problem is a sensor related estimation problem, we aim to solve it using *only* the information retrieved from a *laser range finder*, or *LIght Detection And Ranging* (LIDAR).

The problem has been widely investigated in literature, for example fusing a LIDAR with a vision system ([2], [3]), or using independently moving platforms for laser scanners ([4], [5]). Our framework comprises a mobile platform equipped *only* with a rigidly fixed LIDAR whose plane of measurements is parallel to the plane of motion. Such a choice is motivated by the intrinsic mechanical robustness (since one of its application is related to the DARPA Grand Challenge competition) and by the limited computational time constraint (a moving LIDAR frame has several plane of measurements that should be processed at each time step). Moreover, a slightly different definition of localization is considered here, since the agent does not have an a–priori map of the environment, and it is not a proper SLAM problem, since a complete map of the surroundings is not needed. Therefore, the problem we are trying to solve is the pure localization in an unknown environment, starting from a well defined initial position (usually the global reference frame is fixed to the environment and coincident with the initial position of the moving robot, customary in the SLAM literature). To maintain positions information over time, an Extended Kalman Filter (EKF) is proposed and compared to an existing particle filter approach, originally presented for face tracking.

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A. Algorithm Components

The measurements collected from the laser scanner are generically referred in literature as *point clouds*, or *range images*, collections of three–dimensional points that belong to the scene in view. An effective localization algorithm that profitably uses a laser scanner has to retrieve motion information from the sensed point clouds, in other words it has to solve the *point registration*, (or *absolute orientation problem*). The registration is generically defined for two point clouds and reconstructs the relative positions from which two measurements of the same "scene" are taken (localization) and/or the geometry of the scene in view (SFM or mapping problem).

In literature a lot of registration algorithms have been presented (e.g. see [6] or the *Maximum Likelihood Estimation Sample Consensus* (MLESAC), proposed in [7]). A distinction among the proposed algorithms is related to the features adopted: surfaces, lines ([8]) or points. Most often, the point–based algorithms are used, like the well–known *Iterative Closest Point* (ICP) algorithm ([9]) and its robust variants ([10]). Instead of computing a point–to–point association, like the ICP does, and then minimizing over the sum of association errors, other authors propose the use of global cost indices over the point distributions using kernel correlation ([11]) or Gaussian mixture densities alignment ([12]).

All the approaches presented in literature solve the registration problem between two set of points, dealing with partially overlapping scenes or noise or outliers. Nevertheless, for the best of the author's knowledge, there is not yet a robust solution with a particular attention to the execution time for an on–line feasible robotic application. Hence, this paper presents a very simple solution to the registration problem based on the *RANdom SAmple Consensus* algorithm (RANSAC, [13]) and on the *Huber kernel* ([14]) that is both quite accurate and fast.

With very little increment in complexity, the ability of the RANSAC algorithm to generate hypotheses and then voting them is also used for dynamic environments. Indeed, when portions of the scene in view have relative motions but there is a lack of highly identifiable features in the sensor readings, it is very difficult to uniquely identify each portion of the scene without using multiple hypotheses (see [15]). In the presented paper, since the RANSAC–based algorithm is used to detect rigidity, it is iterated to estimate all the possible rigid transformations in the sensed environment, with minimal changes in the implementation, in order to preserve its robustness and its efficiency.

II. THE ABSOLUTE ORIENTATION PROBLEM

The measured data acquired by sampling the same scene at different time instants and from different perspectives, is expressed in different coordinate frames. Registration is the process of coherently expressing the different sets of data into one coordinate system, hence it is a basic component of a localization algorithm.

The sensor adopted in this paper, a laser scanner, emits a laser beam at angle θ and measures the time–of–flight of the beam till it hits a surface in the environment and returns to the source. From the wave length of the laser beam and the time–of–flight it is possible to reconstruct the distance of the surface hit. Hence the measurements available are the angle of emission of the beam θ and the distance ρ to the surface. Each beam of the LIDAR is generated from the origin of the LIDAR frame O_l with an angle value $\theta_i \in [0,\pi]$, where the angle is measured from the Y_l axis (hence, the beam at angle $\theta_i = 0$ lies on the Y_l axis with the same versus, the beam at angle $\theta_i = \pi/2$ lies on the X_l axis with the same versus). With this assumptions, each measurement is a point $p_i = [x_i\, y_i]^T \in \mathbb{R}^2$ in the plane of measurements $\overline{X_l Y_l}$, hence expressed in the LIDAR frame $< L >$ with $x_i \geq 0$. Each measured point has a three-dimensional representation $P_i \in \mathbb{R}^3$, where $P_i \in \overline{X_l Y_l}$. More precisely, consider a fixed world frame $< W >$ and two generic LIDAR's positions, which correspond to two frames $< L_1 >$ and $< L_2 >$. Let the coordinates of the world points P_i, $\forall i = 1,\dots,n_w$, w.r.t. $< L_1 >$ and $< L_2 >$ be ${}^{l_j}P_i = \bar{g}({}^{l_j}\bar{R}_w, {}^{w}\bar{T}_j)^{w}P_i$, where the generic $\bar{g}(\bar{R},\bar{T}) \in SE(3)$. Therefore ${}^{l_2}P_i = \bar{g}({}^{l_2}\bar{R}_{l_1}, {}^{l_1}\bar{T}_2)^{l_1}P_i$.

Let us now define the mapping function $\Gamma : \Upsilon_k \subset \mathbb{R}^3 \rightarrow \mathbb{R}^2$ of each point in the 3–dimensional world ${}^{w}P_i \in \mathbb{R}^3$ onto the LIDAR measurement space with reference frame $< L_k >$

$${}^{l_k}p_i = \Gamma_k({}^{w}P_i)$$

where the set $\Upsilon_k = \{{}^{w}P_i \in \mathbb{R}^3 | {}^{l_k}\bar{R}_w({}^{w}P_i - {}^{w}\bar{T}_k) \cdot [001]^T = 0\}$ and ${}^{w}\bar{T}_k$ is the translation vector that connects the origin of $< W >$, O_w, with O_{l_k}, expressed in $< W >$. Hence

$${}^{l_k}p_i = \Pi^{l_k}\bar{R}_w({}^{w}P_i - {}^{w}\bar{T}_k),\ \forall P_i \in \Upsilon_k \subset \mathbb{R}^3$$

where Π is the canonical projection matrix from 3–D to 2–D.

Consider now the set of 3–D points ${}^{w}P_i \in \Upsilon_1$ and ${}^{w}P_i \in \Upsilon_2$, with $i \in I_w$ is the index of a sensed point that is mapped both onto $< L_1 >$ and $< L_2 >$. Hence, $\forall i \in I_w$ there exists a $j \in I_{l_1}$ and a $k \in I_{l_2}$ such that ${}^{l_1}m_j = \Gamma_1({}^{w}P_i)$ (m_j stands for the j–th *model point*) and ${}^{l_2}s_k = \Gamma_2({}^{w}P_i)$ (s_k stands for the k–th *scene point*). Therefore, $\#I_w = \#I_{l_1} = \#I_{l_2}$ and $\forall j \in I_{l_1}$ there exists $k \in I_{l_2}$ (i.e. *correspondence problem*) such that ${}^{l_2}s_k = g({}^{l_2}R_{l_1}, {}^{l_1}T_2)^{l_1}m_j$, where $g(R,T) \in SE(2)$. In practical applications the set Υ_k should be substituted with Υ_{m_k}, that is the set of measurable points in the k–th position taking into account the limited angle of the LIDAR ($\theta_i \in [0,\pi] \Rightarrow p_i \cdot X_l > 0$) and its limited range ρ_i ($\rho_i \leq \rho_{max} \Rightarrow \|p_i\|_2 \leq \rho_{max}$). Moreover, Υ_{m_k} should be further modified taking into account the visibility constraint

$$\Upsilon_{v_k} = \{P_i \in \Upsilon_{m_k} | \forall Y \in \Upsilon_{m_k} s.t. Y = O_{l_k} + d({}^{w}P_i - O_{l_k}),\ d \geq 1\}$$

With this model, the problem of registration between two scans can be defined as: **Given the set of points** ${}^{w}P_i \in \mathbb{R}^3$**, with** $i \in I_w$**, such that** ${}^{l_1}m_j = \Gamma_1({}^{w}P_i)$**, with** $j \in I_{l_1}, {}^{w}P_i \in \Upsilon_{v_1}$**, and** ${}^{l_2}s_k = \Gamma_2({}^{w}P_i)$**, with** $k \in I_{l_2}, {}^{w}P_i \in \Upsilon_{v_2}$**, find the transformation** $g(R,T) \in SE(2)$ **between the sets** $v_{m_1} = \{m_j, \forall j \in I_{l_1}\}$ **and** $v_{m_2} = \{s_k, \forall k \in I_{l_2}\}$.

Notice that with the additional assumption of plane of measurements constantly parallel to the plane of motion of the LIDAR (assumption easily verified for indoor environments or factory floors, as well as the majority of the urban roads) and solving the registration problem for transformation in $SE(2)$, the robot is localized in the three–dimensional, motion–less world frame $< W >$ further assuming that in the first position the rigid transformation $\bar{g}({}^{w}\bar{R}_{l_1}, {}^{w}\bar{T}_1)$ is known (in practice, $\bar{g}({}^{w}\bar{R}_{l_1}, {}^{w}\bar{T}_1) = \bar{g}(I_3, [0,0,0]^T)$).

III. RANSAC BASED REGISTRATION

The first problem we are trying to solve in this paper is to find an accurate transformation between two point sets in the presence of noise and outliers, preserving convergence efficiency. The very basic idea of our solution relies on the RANSAC algorithm, which fits a model to the randomly–sampled experimental data and interprets/smooths measurements containing a significance percentage of gross errors. Since we are interested in the sensor localization rather than in the environment mapping, we try to robustly search for "rigidity" in the available data (see [16] for a similar approach). More precisely, given the model set $\mathscr{M}$ and the scene set $\mathscr{S}$ of measured points taken respectively from frame positions $< L_1 >$ and $< L_2 >$, RANSAC tries to robustly estimate the rigid transformation $g(R,T) \in SE(2)$ that relates the two positions.

Our algorithm is very simple and it is depicted in details in Table I. Notice that the rigid transformation is used to determine the *inliers*, i.e. the points that verify the hypothesis. In an ideal case where there is no noise, no outliers and the point correspondence is correct, this rigid transformation is also the actual rigid transformation that connects the two point clouds. In the presence of nuisances, robustness is added computing the registration using a least squared estimate on the inliers (see point $4-c$ in Table I).

Following [13], the algorithm is applied for a fixed number of iterations k that depends on the expected level of noise and outliers percentage and guarantees to select a valid couple. More precisely, k is function of the desired probability w to select correctly n data points, the minimum number of points to estimate the model. In our case $n = 4$, 2 points for each measured set. The number k is estimated experimentally. For instance, in the presence of noise with 30% of outliers and

1) **Selection of the *scene control points***
   (a) Randomly select a couple of points s_1 and s_2 in the set $\mathscr{S}$ and compute their distance $d_{\mathscr{S}} = \|s_1 - s_2\|$.
2) **Selection of the *model control points***
   (a) Randomly select in $\mathscr{M}$ a couple of points (m_1, m_2) with the constraint $d_{\mathscr{M}} = d(m_1, m_2) \simeq d_{\mathscr{S}}$ (i.e. such that $|d_{\mathscr{S}} - d_{\mathscr{M}}| \le d_{th}$, where d_{th} is a user defined threshold). If such a couple does not exist, go back to the step 1.
3) **Model Parameter Estimation**
   (a) Find the transformation parameters R and T using the selected couples: $\phi = \angle(s_1 - s_2) - \angle(m_1/m_2)$ and $T = 1/2(s_1 + s_2 - R(\phi)(m_1 + m_2))$.
   (b) Apply the estimated transformation to the set $\mathscr{S}$: $\mathscr{S}' = g(R,T)\mathscr{S}$.
4) **Model Verification**
   (a) Count the number N_{in} of points in $\mathscr{S}'$ consistent with the model $\mathscr{M}$ (inliers).
   (b) Go back to step 1 until a maximum number of iterations is reached.
   (c) - Select the hypothesis with the largest number of inliers and solve the least mean square problem $(R,T)^* = \arg\min_{(R,T)} \sum_{i=1}^{N_{in}} \|m_i - g(R,T)s_i\|^2$ only on the found inliers and return $(R,T)^*$.
       - If $N_{in} < N_{min}$, return with no results.

TABLE I
A SUMMARY OF THE STAGES OF THE RANSAC–BASED ALGORITHM FOR A 2–D TRANSFORMATION.

a rotation angle $\phi = \pi/8$, we might expect to find a correct correspondence in $k \approx 53$ selections.

It is worthwhile to note that similarly to our robust solution of successive measurements association (via rigidity), [17] use an effective hypotheses and validation scheme for data association, although robustness is achieved if an accurate sensor location and an a–priori map of landmarks are available. Furthermore, the outliers are explicitly considered in that work only in the scene $\mathscr{S}$ but not in the model (map) $\mathscr{M}$.

A. Comparative Examples in the Ideal Case

The RANSAC–based algorithm presented in the previous section is compared with some of the more efficient registration algorithms presented in literature. To make the comparison more fair (the algorithms in literature are not explicitly thought for LIDAR measurements but for generic point clouds), an ideal sensor is used in this section, whereas there is no angle limitation $\theta_i \in (-\pi, \pi]$, the angle distance between two consecutive beams $\theta_{i+1} - \theta_i = \varepsilon > 0$ is little as desired, the range measurements are theoretically infinite $\rho_{max} = +\infty$ and there is no occlusions in the measurements. Therefore, considering a set of K LIDAR positions w.r.t. the reference frame $< W >$, $\Upsilon_{v_k} \equiv \Upsilon_{m_k} \equiv \Upsilon_k$ holds $\forall k \in K$. Moreover, each of the measured point is isolated, that is given two measured points $p_i, p_j \in \mathbb{R}^2$, $\|p_i - p_j\| > \delta$, where δ is the standard deviation of the Gaussian distributed noise.

The algorithms used for the comparisons are the ICP (**??**), the Kernel Correlation ([11]), and the *Coherent Point Drift* ([12]). In figure 1 the accuracy on the X_w and Y_w axis are

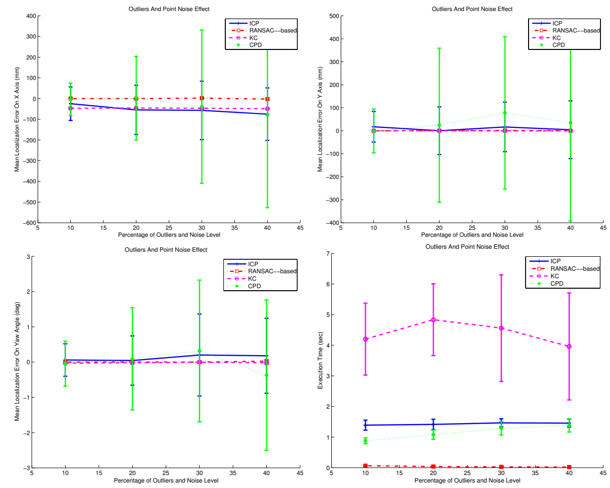

Fig. 1. Accuracy on the X_w and Y_w axis (in millimeters, upper row), accuracy of the orientation angle (in degrees, bottom row, left) and the execution time (in seconds, bottom row, right). The effect of both noise (drawn from a Gaussian distribution with increasing standard deviation from 0*mm* to 40*mm*) and outliers (with a percentage from 0% to 40%) are reported.

reported in the upper row (in millimeters), while in the bottom row the accuracy of the orientation angle (reported in degrees) and the execution time (in seconds) are depicted. The effect of both noise (drawn from a Gaussian distribution with increasing standard deviation from 0 mm to 40 mm) and outliers percentage (from 0% to 40%) are reported. In the graphs, the mean error and its standard deviations are reported for the accuracy and for the execution time. Notice that a percentage of 40% of outliers both in the scene and in the model point clouds represent a total percentage of 80% in the worst case. As it can be easily seen, the RANSAC–based algorithm performs better than the other algorithms in accuracy and is considerably faster to be executed.

IV. STRUCTURED ENVIRONMENTS

The characteristics of the environment are relevant for the registration problem, since it involves pure geometric information. Hence, the geometry of the environment needs to be explicitly analyzed to better understand its impact on the accuracy of the algorithm. For simplicity's sake, the surrounding ambient is considered as consisting of generically oriented planes.

Consider two successive measurements of the same surface taken from different positions. Since the LIDAR measures the three–dimensional points using a single source for the laser beam, but with different orientation $\theta_i \in [0, \pi]$, the measurement process is equivalent to map the surrounding ambient using a polar–distributed grid. Hence, the sensed points (i.e. points on the grid) from two successive measurements of the same surface have no correspondence in general, in fact each point "slides" on the surface of an amount that depends both on the proximity of the sensed surface from the emitter and on the trajectory of the sensor. Furthermore, from geometric observations, also the density of the measured points depends on the relative distance of the emitter from the surface.

Due to the simplicity of the presented RANSAC–based registration algorithm, it is possible to introduce some constraint in the randomized point selection to reduce the effect of the noise and the absence of a reliable correspondence in the measurements. Let us consider the LIDAR measurements at time t generated according to the model $p_i(t) = f(\alpha(t), n(t))$, where $\alpha(t)$ is the parameter vector (that comprises the position of the LIDAR and the position of the surfaces in the world) and $n(t)$ is the measurement noise. Without loss of generality, assume that the uncertainty on the measurements can be expressed, according to the Gauss Error Theory Model ([18]), as an additive noise $p_i(t) = f(\alpha(t)) + n(t)$, where $f(\alpha(t))$ is the ideal data measurement function. Let's assume the Central Limit Theorem is valid in this contest and that the instrument is not affected by systematic errors. Then we can think that $n(t)$ is normally distributed with zero mean and Σ variance, $n(t) \sim \mathcal{N}(\mathbf{0}, \Sigma)$ where

$$\Sigma = \begin{bmatrix} \sigma_\rho^2 & \sigma_{\rho\theta} \\ \sigma_{\theta\rho} & \sigma_\theta^2 \end{bmatrix}$$

Furthermore, assume that the error along the beam's direction ρ is independent from the error along the angle θ and that $\sigma_\rho >> \sigma_\theta$ (assumed quite often in the LIDAR technical references). Assuming that three points are measured from the same plane in the space, recalling that the uncertainty is given by σ_ρ and $d \sin(\tilde{\theta}_d) \approx 2\sigma_\rho$, where $\tilde{\theta}_d$ is the orientation of the surface w.r.t. to the laser scanner reference frame and d is the distance between two points, the more the distance d is increased the more the orientation error is decreased. Therefore, the RANSAC–based algorithm will firstly try to select far apart points to improve the accuracy of the orientation estimate.

Notice that, in the case of a single measured plane whose dimension is bigger than ρ_{max}, all the measured points lie on a single lines for two successive measurements. Hence, only the translations that are perpendicular to the plane are recovered, while the translations parallel to the plane are arbitrary. This observation holds also in the case of parallel planes.

Due to the aforementioned surface point sliding, a correct correspondence does not exist in general. Since our algorithm is related to the point–to–point association, the voting function of the RANSAC algorithm is modified adopting a Huber kernel, a robust kernel usually used for unknown density estimations. Using the voting schema of RANSAC but applying the Huber loss function $\mathcal{L}_h$

$$\mathcal{L}_h(e_i)^2 = \begin{cases} \frac{1}{2} e_i^2 & \|e_i\| \le t_e \\ \frac{1}{2} t_e (2\|e_i\| - t_e) & \|e_i\| > t_e \end{cases}$$

where $e_i = s_i - m_j$ is the error of the hypothesized correspondence $i \leftrightarrow j$ and t_e is a user defined threshold (i.e. the allowable error distance for an inlier), the computational burden is basically unmodified while a sort of smoothing in the correspondence problem is applied to the data.

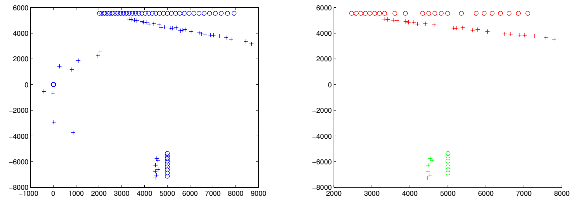

Fig. 2. Left: world configuration of the measured set $\mathcal{M}$. Right: world segmentation and reconstruction using the proposed algorithm. The distances are reported in millimeters.

For space limits, the comparisons between the Huber–modified algorithm and the previously presented comparative algorithms is not reported for the case of measurements with nuisances, even though our solution performs better in this case as well.

A. Extension to Dynamic Environments

One of the most common assumption for registration algorithms regards the absence of portion of the scene that moves independently to each other. Relaxing this assumption opens a lot of new problems that are closely related to the sensor used, since highly identifiable objects in the sensor readings allow a more efficient solution in dynamic environments (e.g. segmentation).

One way to tackle the problem is to exploit the geometric information read from the sensor. Consider a scene sensed from two different positions 1 and 2 and the relative point sets $\mathcal{M}$ and $\mathcal{S}$ and suppose that g_a is the hypothesis with the larger support $\mathcal{S}_a$. The transformation g_a is then considered as the relation between $< L_1 >$ and $< L_2 >$, or, equivalently, that the rigid scene points $\mathcal{S}_a$ are static. Suppose $\#\mathcal{S}_{b_\star} = \#[\mathcal{S} / \mathcal{S}_a] \ge t_{min}$ ($t_{min} > 2$ is the minimum number of points for a valid set, depending on the sensor granularity) and $\#\mathcal{M}_{b_\star} = \#[\mathcal{M} / (g_a^{-1} \mathcal{S}_a)] \ge t_{min}$: a new hypothesis g_{b_1} is generated by the RANSAC–based algorithm, identifying the new inliers scene set $\mathcal{S}_{b_1}$. This step is iterated until $\#\mathcal{S}_{b_\star} = \#[\mathcal{S} / (\mathcal{S}_a \bigcup_{i=1}^n \mathcal{S}_{b_i})] < t_{min}$. Hence, the relative motions $g_a - g_{b_i}$ are computed ($\mathcal{S}_{b_i}$ is often referred as a *pseudo–outliers* set). It is worthwhile to note that this algorithm works quite well if the configuration of the space and the noise amount allows a well defined separation between the two regions, otherwise two measurements are not informative enough to segment without any prior on the point distribution.

In figure 2 a simulation result with the Huber kernel RANSAC algorithm has been reported, using two planes corrupted by noise (whose standard deviation is 100 mm) and with 20% of outliers.

V. EXTENDED KALMAN FILTER

The registration algorithm presented so far is not sufficient to efficiently track the LIDAR trajectory over time. In fact, it only robustly computes the registration $g(R, T)$ between two measurement sets. Hence, using the proposed algorithm alone it is only possible to integrate over time the various transformation estimates, with trivial problems of solution drifts (this is the same problem that arises using only odometric data for mobile robots). A solution in this case is to use a Bayesian based probabilistic description using the laser measurements, implementing the estimation process with an EKF and maintaining a rough point based map for both inliers and outliers (basically, the measured points in a position).

The EKF is a wide popular way to estimate the internal state of a system given the motion prior and noisy measurements. As a matter of fact, since the Kalman filter is not completely resilient to outliers (due to its iterative strategy based on the Maximum A Posteriori), even in its robust version ([19]), we apply RANSAC directly on the measurements to avoid outliers. Indeed, the multi–modality of the conditional density is only due to the output statistics: assuming that the posterior density of the hidden variables (the state of the system related to the registration) is unimodal, the EKF with RANSAC attempts to estimate the principal mode of the dynamic evolution.

A rather similar approach is the KALMANSAC ([20]), where an explicit estimation of the process related to the outliers is added. Nevertheless, in the absolute orientation problem we are not interested in an explicit estimation of the range data but in the localization (or "relocation") of the LIDAR. Hence, an explicit estimation of the range data is not necessary and saves computational time.

More precisely, the continuous time dynamic of the system is easily expressed as

$$\begin{cases} {}^{w}\dot{X} &= {}^{w}\eta \\ {}^{w}\dot{\eta} &= {}^{w}\alpha \\ {}^{w}\dot{p} &= 0 \end{cases}$$

where ${}^{w}X = [x, y, \phi]^T$ represents the actual position of the LIDAR in $< W >$, ${}^{w}\eta = [\eta_x, \eta_y, \eta_\phi]^T$ represents the velocities of the LIDAR, ${}^{w}\alpha = [\alpha_x, \alpha_y, \alpha_\phi]^T$ are the input accelerations (supposed to be unknown in the filter). The vector $p \in \mathbb{R}^{2n}$, ${}^{w}p = [x_1, y_1, \ldots, y_n]^T$ is the vector of all the features measured at the first time instant. The measurement functions h_i, $\forall i = 1, \ldots, n$, are represented by the inlier positions given by the RANSAC–based registration algorithm

$${}^{l_t}p_i(t) = h_i(X, \eta, p(0)) = g(R(\phi), T(x, y))\,{}^{w}p_i(0)$$

(i.e. $< L_0 > \equiv < W >$). Using element of nonlinear control theory ([21]), the complete observability of such simple model has been demonstrated if the initial position of the LIDAR w.r.t. the fixed frame $< W >$ is known.

A major drawback of this solution is the filter hacking as soon as there are not enough common points between the current and the initial measurement sets. More in depth, suppose that at time $\bar{t}$ there are not enough inliers given by the registration algorithm: the point stored from the initial position ${}^{w}p(0)$ are substituted by ${}^{l_{\bar{t}-1}}p(\bar{t}-1)$, and the measurements will be

$$\begin{aligned} {}^{l_{\bar{t}}}p_i(\bar{t}-1) = h_i(X, \eta, p(\bar{t})) = g(R(\phi), T(x, y)) \\ \hat{g}^{-1}(R(\hat{\phi}), T(\hat{x}, \hat{y}))^{l_{\bar{t}-1}}p_i(\bar{t}-1) \end{aligned}$$

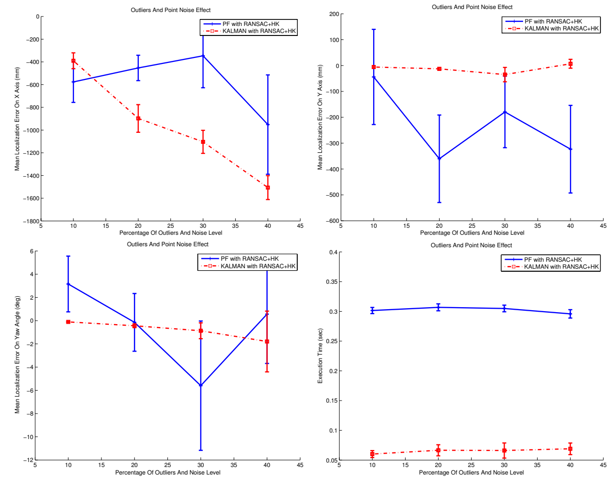

Fig. 3. Accuracy on the X_w and Y_w axis (in millimeters, upper row), accuracy of the orientation angle (in degrees, bottom row, left) and the execution time (in seconds, bottom row, right). The effect of both noise (drawn from a Gaussian distribution with increasing standard deviation from $0mm$ to $40mm$) and outliers (with a percentage from 0% to 40%) are reported.

where $\hat{g}$ is the transformation between $< W >$ and $< L_{\bar{t}-1} >$, and hence ${}^{w}\hat{X} = [\hat{x}, \hat{y}, \hat{\phi}]^T$ is the estimate till time $\bar{t}-1$, not further updated (unless a closed loop is detected).

The computational burden of this approach needs some more considerations to be qualitatively evaluated. Indeed, the Kalman–state update step is critically related to the dimension of the state space to estimate, that is $6+2n$. In practice, the number of inliers stored in the state space is just a portion (typically the 60%) of the total number of measurements (usually in the magnitude order of few hundreds), so the overhead introduced is negligible on modern computers.

A. *Simulation Comparisons with RANSAC–PF*

Among the robust algorithms for robot localization, the solution proposed by [22] has been chosen for comparison, a particle filter approach for face tracking (that can be easily extended to localization). In this approach, called RANSAC-PF, the feature correspondences (or the registration itself) is used to generate state hypotheses using RANSAC for the estimates. Strictly speaking, in the EKF approach presented above, the RANSAC algorithm is used to generate a measurement of the inliers that, weighted with the Kalman gain, corrects the prediction, that is equivalent to the generation of a new particle in the RANSAC–PF, but with clear advantages in the accuracy of the measurement model involved. Indeed, in RANSAC–PF only a subset of the measurements, strictly necessary to the new state generation, is used and, furthermore, it is assumed that the model is completely observable in one step.

The simulation results reported in figure 3 are referred to a generic point cloud measured from several different positions assuming an ideal sensor. The simulation parameters are the same of the previously presented results in section III-A. Even though the accuracy is quite similar between the two

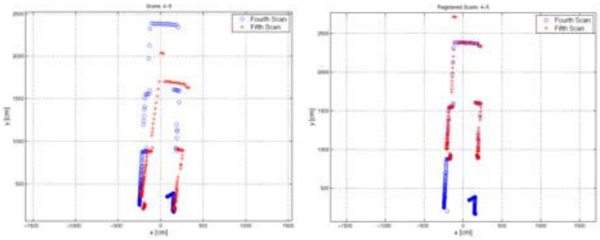

Fig. 4. Left: two consecutive scans of the corridor experiments. Right: correctly registered scans.

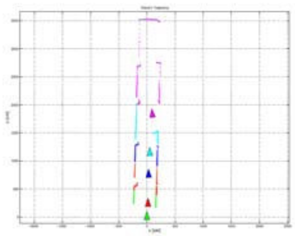

Fig. 5. Final registered map.

estimation processes presented (nearly 5% of the travelled path, both for position and orientation), the KALMAN and RANSAC algorithm, in combination with the Huber Kernel, shows an evident improvement with respect to the execution time, salient characteristic for on–line applications.

VI. EXPERIMENTAL RESULTS

In this final section some experimental results that validate the feasibility of the presented approach are presented. Five measurements along the Computer Science Department corridor at UCLA has been collected with a LMS 291 SICK laser scanner mounted on a moving platform. In figure 4 are shown the single registration results for two consecutive measurements while the figure 5 depicts the overall result of the map reconstruction by composition of the registered inliers measurements and of the robot pose estimate after five sequential scans.

VII. CONCLUSIONS

In the presented paper the problem of accurate localization for a moving LIDAR sensor has been presented. Particular attention has been devoted to the execution time of the algorithm since it can limits its on–line application. A fast and accurate registration algorithm is presented, which is able to deal with noisy measurements and large amounts of outliers. The main idea of this work is to make the most of the well known RANSAC algorithm in combination with a Huber kernel. To show the effectiveness of the proposed algorithm, a comparison with some of the solutions presented in literature for robust registration has been presented.

For a practical application of the algorithm, a combination with an Extended Kalman Filter is also presented in order to continuously estimate the trajectory followed by the LIDAR, without using odometry for generality.

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